Enrollment No: ______________________ Exam Seat No: ______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name: Group Theory

Subject Code: 5SC02GRT1 **Branch: M.Sc. (Mathematics)**

Semester: 2 **Date: 12/05/2017** **Time: 02:00 To 05:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Every finite group of order less than six is abelian. True/False. **(01)**

b. Define: Index of subgroup of group. **(01)**

c. Define: Normalizer of element of group. **(01)**

d. Prove that every cyclic group is an abelian group. **(02)**

e. If $a^2 = e$ for all element of a group G. Then prove that G is commutative. **(02)**

Q-2 **Attempt all questions** **(14)**

a) If $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} \, / \, a \text{ is non zero real number} \right\}$ then prove that G is group under multiplication. **(05)**

b) If $a \in G$ is of order n then prove that $a^m = e$ for some integer m if and only if $n \mid m$. **(05)**

c) If G is finite group of order n and $a \in G$ then prove that there exists a positive integer $r \leq n$ such that $a^r = e$. **(04)**

OR

Q-2 **Attempt all questions** **(14)**

a) If $(G,*)$ be a group and $o(a) = n$ then prove the following **(06)**
(i) $o(a^q) \leq o(a)$ for any integer q.
(ii) $o(a^{-1}) = o(a)$.

b) Let $G = \{(a,b) \mid a,b \in R,\ a \neq 0\}$ and $*$ be defined as $(a,b) * (c,d) = (ac, bc + d)$ then prove that G is a group under $*$. **(05)**

c) If $(G,*)$ is a group and $a, b \in G$ then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ **(03)**

Q-3		**Attempt all questions**	**(14)**
	a)	If H is a subgroup of a finite group G then prove that $o(H) \mid o(G)$.	**(06)**
	b)	Prove that a group of prime order is cyclic.	**(04)**
	c)	Define: Centre of group G. Prove that Centre of group G is subgroup of G.	**(04)**

OR

Q-3	**a)**	If in a group $G,\ a^5 = e, aba^{-1} = b^2$ for $a, b \in G$find o(b).	**(06)**
	b)	Prove that product of two right cosets is a right coset.	**(04)**
	c)	If H and K are normal subgroups of a group G with $H \cap K = \{e\}$ then prove that $kh = hk$ for each $h \in H$and $k \in K$.	**(04)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	If $\emptyset: G \to G'$ be a homomorphism with $K_{\emptyset} = \{e\}$ then prove that $\emptyset$ is one – one.	**(02)**
	b.	How many generators are there of the cyclic group of order 8?	**(02)**
	c.	Define: p - sylow group.	**(01)**
	d.	Define: Internal direct product.	**(01)**
	e.	Define: Transposition	**(01)**

Q-5		**Attempt all questions**	**(14)**
	a)	If $\phi : (G, \cdot) \to (G', *)$ is an onto homomorphism with kernel K then prove that $G \mid K \cong G'$.	**(06)**
	b)	Let G be group. For a fixed element $g$in G, define $\phi : G \to G$ by $\phi(x) = gxg^{-1}$. Prove that is ϕ an isomorphism of G on to G.	**(05)**
	c)	Write all the permutations on three symbols 1, 2, 3. Which of these are even permutation?	**(03)**

OR

Q-5	**a)**	Let G be a group and suppose G is the internal direct product of $N_1, N_2, \dots, N_n$. Let $T = N_1 \times N_2 \times \dots \times N_n$. Then prove that G and T are isomorphic.	**(07)**
	b)	Prove that each permutation $f \in S_n$ can be expressed as a composition of disjoint cycles.	**(07)**

Q-6		**Attempt all questions**	**(14)**
	a)	Prove that every finite group G is isomorphic to a permutation group.	**(07)**
	b)	Suppose $G_1, G_2, \dots, G_n$ are groups. Let $G = G_1 \times G_2 \times \dots \times G_n$. Let $a = (a_1, a_2, \dots, a_n)$ and $b = (b_1, b_2, \dots, b_n) \in G$. The binary operation $*$ defined as $a * b = (a_1b_1, a_2b_2, \dots, a_nb_n)$. Prove that $(G,*)$ is group.	**(07)**

OR

Q-6	**a)**	State and prove Sylow's theorem.	**(07)**
	b)	If G is a group then prove that $\mathcal{A}(G)$, the set of all automorphism of G is also a group under the composition of functions.	**(07)**